THE MATH CIRCLE

O-LEVEL

Paper 1

Locus

WorkSheet

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Q1 (this is Redspot Q#)

Loci and Construction

For Examiner's Use

For Examiner's Use

20 The diagram in the answer space is a map showing a section of coastline and a beacon on land.
Fishing boats can only operate when they are

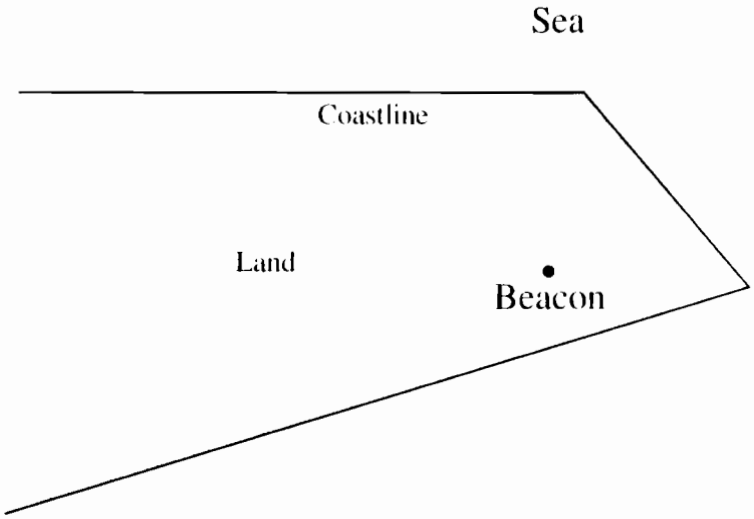
I not more than 6.5 km from the beacon,

II at least 2 km from the coastline.

The scale of the map is 1 cm to 1 km.

Construct the boundaries of the region where fishing can take place.
Label this region *F*.

Answer



[4]

Q2 Loci and Construction

For Examiner's Use

For Examiner's Use

24 In triangle ABC, $AB = 15$ cm, $BC = 8$ cm and $AC = 11$ cm.
The side AB is drawn in the answer space.

(a) Using ruler and compasses only, complete the triangle.

(b) Measure the largest angle of the triangle.

(c) Draw the locus of **all** points **within the triangle** that are

(i) 5 cm from C,

(ii) equidistant from BA and BC.

(d) The point P, within the triangle, is such that $PC = 5$ cm and P is equidistant from BA and BC.

Label the point P and measure the distance PA.

Answer (a) and *(c)*

A *B*

[3]

Answer (b)..... [1]

(d) PA = ...cm [1]

For Examiner's Use

Q4

For Examiner's Use

23 The diagram below is a scale drawing representing three coastguard stations, *A*, *B* and *C*. In the drawing, 1 cm represents 20 km.

(a) **(i)** Express the scale in the form 1 : *n*.

(ii) Find the distance between the coastguard stations *A* and *B*.

Answer (a)(i) 1 : ..... [1]

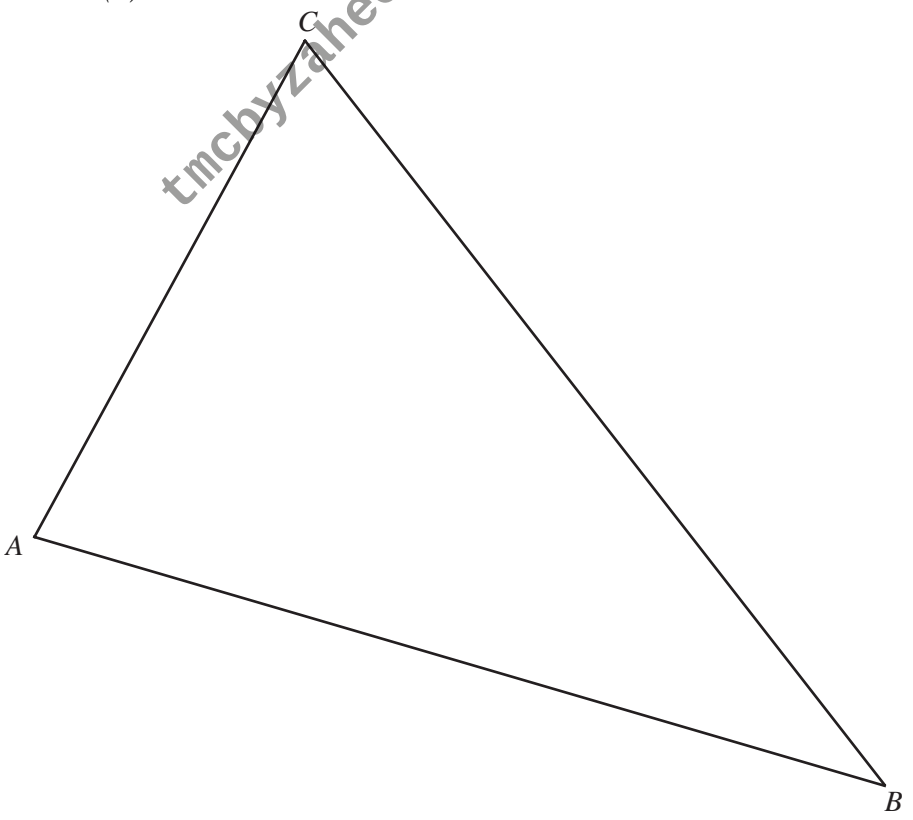
(ii) .....km [1]

(b) It is known that a ship is

I equidistant from *CA* and *CB*,
II nearer to *C* than *A*,
III less than 200 km from *B*.

By constructing 3 loci corresponding to **I**, **II** and **III**, find the possible positions of the ship and label the extreme positions *S* and *P*.

Answer (b)



[4]

For Examiner's Use

Q5

22 A map is drawn using a scale of 1 cm to 5 m.
The position of *A* is shown in the answer space below.

(a) The point *B* is 70 m due East of *A*.
Draw the line representing *AB*.

(b) The point *C* is North of *AB* and equidistant from *A* and *B*.

Angle *BAC* = 40°.

(i) By drawing appropriate lines, find and label the point *C*.

(ii) Find the actual distance *AC*.

(iii) State the size of the reflex angle *BAC*.

Answer (*a*) and (*b*)(i)

[3]

Answer (*b*)(ii) ..... m [1]

(iii) reflex $B\hat{A}C$ = ...[1]

For Examiner's Use

Q6

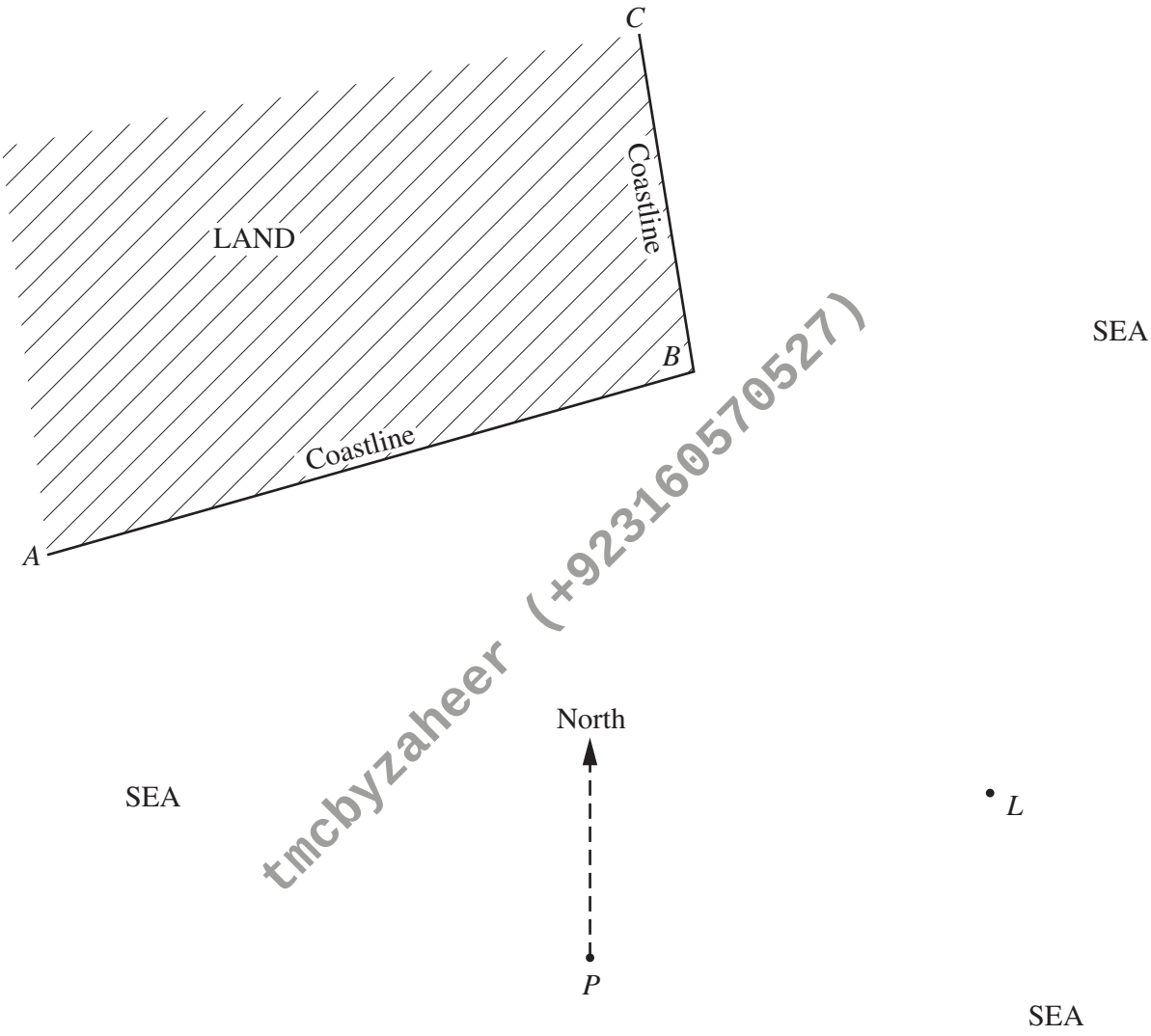
For Examiner's Use

23 The diagram below is a map showing a coastline *ABC*, a lighthouse *L* and a point *P*.
The map is drawn to a scale of 1 cm to 100 m.
Ships must not sail within 200 m of the coastline nor within 200 m of the lighthouse.

(a) Construct the locus of points 200 m from the lighthouse *L*.

(b) Construct the locus of points 200 m from the coastline *ABC*.

Answer (a) (b)



[3]

(c) A ship sailed from the point *P* on a bearing of θ°.
It passed between *B* and *L*.
Complete the statement in the answer space.

Answer (c) ..... < θ < .....[2]

For Examiner's Use

Q7

20 The plan of a field has a scale of 1 cm to 5 metres.

(a) Express this scale in the form 1 : *n*.

Answer (*a*) .....[1]

(b) The plan was made by measuring angles from two points, *A* and *B*, 50 m apart.
The line *AB* is drawn to scale in the answer space below.

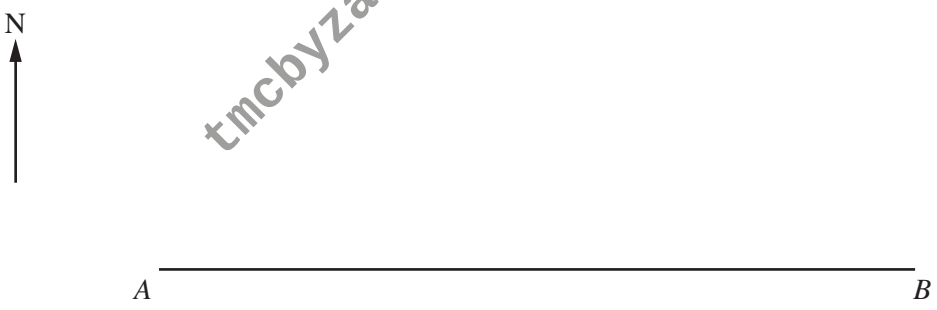
(i) A tree is at the point *T* in the field.
$B\hat{A}T = 35°$ and $A\hat{B}T = 70°$.

Locate and label *T* on the plan.

(ii) Given that *A* is due west of *B*, state the bearing of *T* from *B*.

(iii) By making an appropriate measurement, find the actual distance, in metres, of the tree from *B*.

Answer (*b*)(i)



[1]

Answer (*b*)(ii) ..... [1]

(iii) ..... m [1]

For Examiner's Use

Q8

For Examiner's Use

25 The diagram at the bottom of the page shows a triangle ABC.

(a) By measuring an angle, find reflex angle ACB.

Answer (*a*) .....[1]

(b) The point D is on the opposite side of AC to B.
$AD = 6$ cm and $CD = 9$ cm.
Construct triangle ACD.

(c) On the diagram, construct the locus of points, **inside** the quadrilateral $ABCD$, which are

I equidistant from A and C,

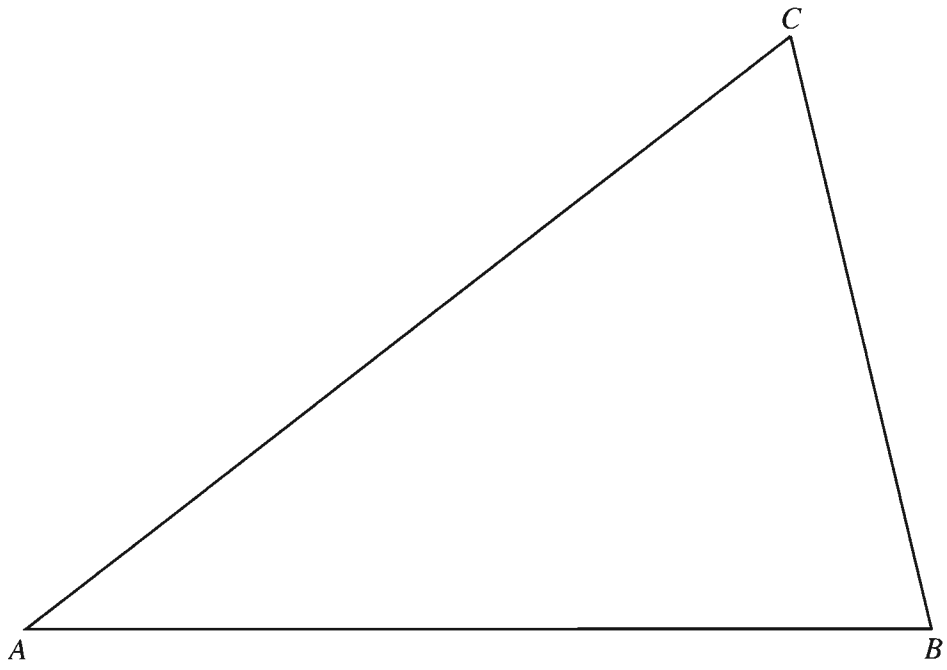
II 5 cm from the line AB.

(d) The point P is inside quadrilateral $ABCD$,
equidistant from A and C,
5 cm from the line AB.

(i) Mark and label the position of P.

(ii) Measure CP.

Answer (*b*)(*c*)(*d*)(i)



[3]

Answer (*d*)(ii) CP = ...cm [1]

For Examiner's Use

Q9

23 A map is drawn to a scale of 1 cm to 3 km.
The diagram below shows the positions of two villages *A* and *B* on the map.

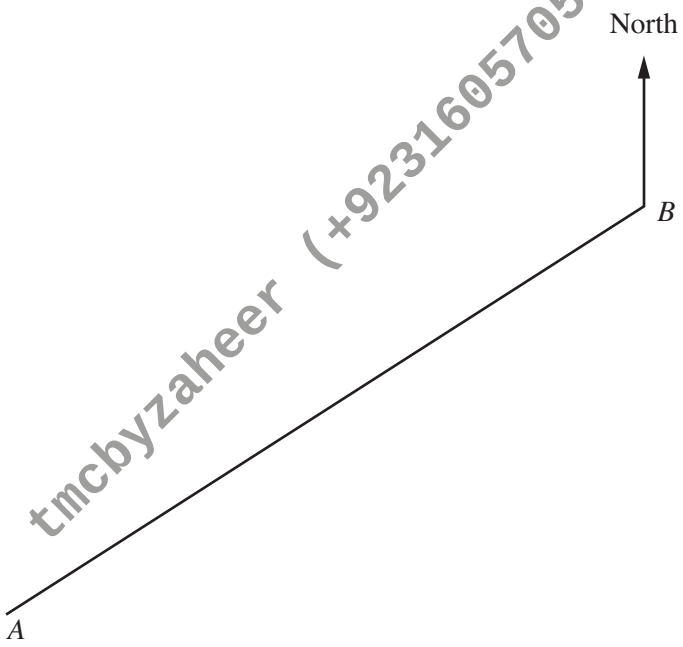
(a) **(i)** Write the scale in the form 1 : *n* .

(ii) Find the actual distance, in kilometres, between the villages *A* and *B*.

Answer (*a*)(i) 1 : .....[1]

(ii) ..... km [1]

Answer (*b*), (*c*), (*d*)



[4]

(b) A third village, *C*, lies north of the line *AB*.
It is 21 km from *A* and 18 km from *B*.

Using ruler and compasses only, construct triangle *ABC*.

(c) Construct the perpendicular bisector of *AB*.

(d) A petrol station is to be built so that it is equidistant from *A* and *B* and 9 km from *C*.

Mark with letters *F* and *G* the two possible positions of the petrol station.

Q10

For Examiner's Use

13 A family wants to move to a new house.
The area where they are going to look depends on the positions of the children's school, S, the father's place of work, F, and the market, M.
The diagram in the answer space is drawn to a scale of 1 cm to 1 km.
It shows the positions of S, F and M.

The house needs to be:

I within 4 km of the children's school,

II nearer to the market than to the father's place of work.

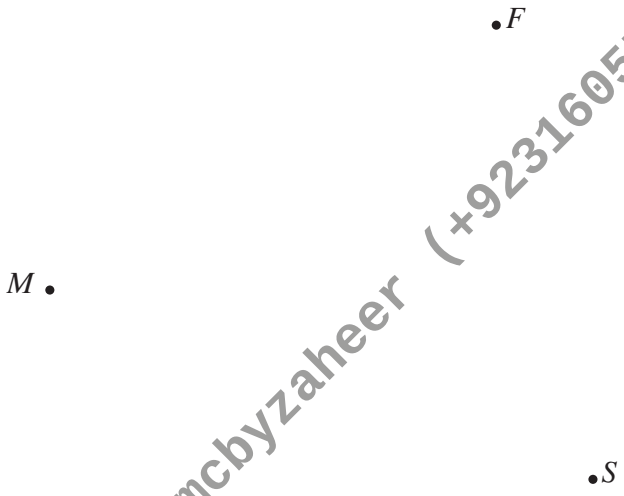
(a) Use I and II to construct the appropriate loci.

(b) Shade the region of your diagram that represents the possible positions of the house.

(c) Find the greatest possible distance between the house and the market.

Answer (*a*) (*b*)

Scale 1 cm to 1 km



[3]

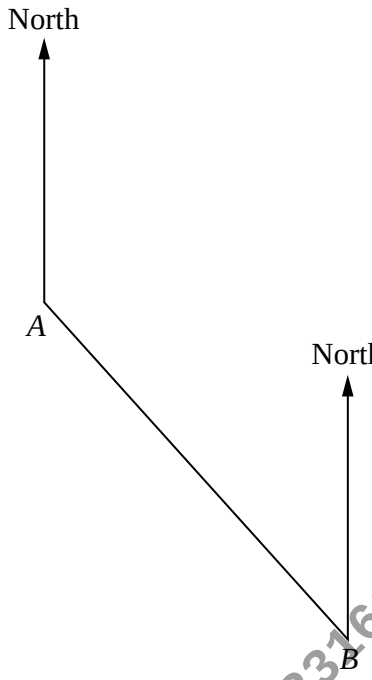
Answer (*c*) ..... km [1]

Q11

23 A sailing club has five moorings in the river at *A, B, C, D* and *E*.
A and *B* are 12 metres apart.
The positions of *A* and *B* are shown in the scale drawing below.

For Examiner's Use

Answer (b), (c), (d)



(a) Write the scale in the form 1 : *n*.

Answer (*a*) 1 : .....[1]

(b) *C* is due west of *B* and on a bearing of 210º from *A*.

Find and label the position of *C*. [2]

(c) *D* lies north of the line *AB* .
The triangle *ABD* is equilateral.

Using ruler and compasses only, construct triangle *ABD*.
Show your construction arcs clearly. [1]

(d) The bearing of *E* from *A* is the same as the bearing of *B* from *A*.

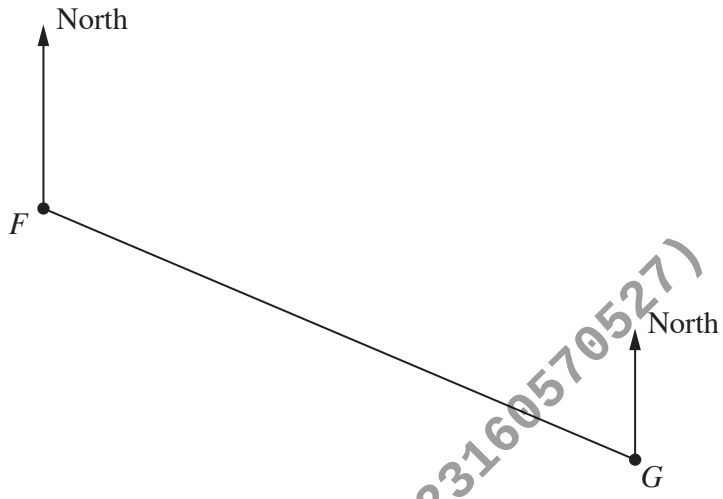
Given that *AB* : *AE* = 3 : 5, find and label the position of *E*. [2]

Q12

16 The scale drawing below shows the positions of two towns, *F* and *G*.
It is drawn to a scale of 1 cm to 3 km.

For Examiner's Use

Answer (*b*)



(a) Find

(i) the distance, in kilometres, between towns *F* and *G*,

Answer (*a*)(i) ..... km [1]

(ii) the bearing of *G* from *F*.

Answer (*a*)(ii) .....[1]

(b) Town *H* is to the North of the line *FG*.
It is 19.5 km from *F* and 15 km from *G*.
On the diagram above, find and label the position of *H*. [1]

Q14

26 A map is drawn to a scale of 1 cm to 5 km.
The diagram below shows the positions of two radio masts *A* and *B* on the map.

For Examiner's Use

Answer (a), (b)

(a) A third radio mast, *C*, is north of the line *AB*.
It is 40 km from *A* and 50 km from *B*.

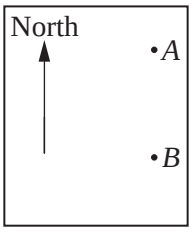
Using ruler and compasses, construct triangle *ABC*. [2]

(b) A house *D*, inside the triangle, is more than 35 km from *B* and closer to *B* than to *A*.

Shade the region on your diagram that represents the possible positions of the house *D*. [3]

Q15

6 Answer this question on a new page.



A and *B* are two coastguard stations with *A* due north of *B*.
On your new blank page, mark *A* in a position near the top right hand corner, as shown in the diagram.

(a) The distance between *A* and *B* is 140 km.

Using a scale of 1 cm to 10 km,

(i) mark the position of *B* and draw the line *AB*, [1]

(ii) construct the locus of the points west of *AB* that are

(a) equidistant from *A* and *B*, [1]

(b) 90 km from *B*. [1]

(b) A ship, *S*, lies to the west of *AB* and is

I nearer to *A* than *B*,

II within 90 km of *B*.

On your diagram, shade the region in which the ship is situated. [1]

(c) It is also known that the bearing of the ship from *A* is 204°.

(i) On your diagram, mark the two extreme positions, S_1 and S_2, of the ship. [2]

(ii) Measure the angle S_1BS_2. [1]

(iii) The bearing of the ship from *B* is $x°$.

Find the least possible value of x. [1]

Q16

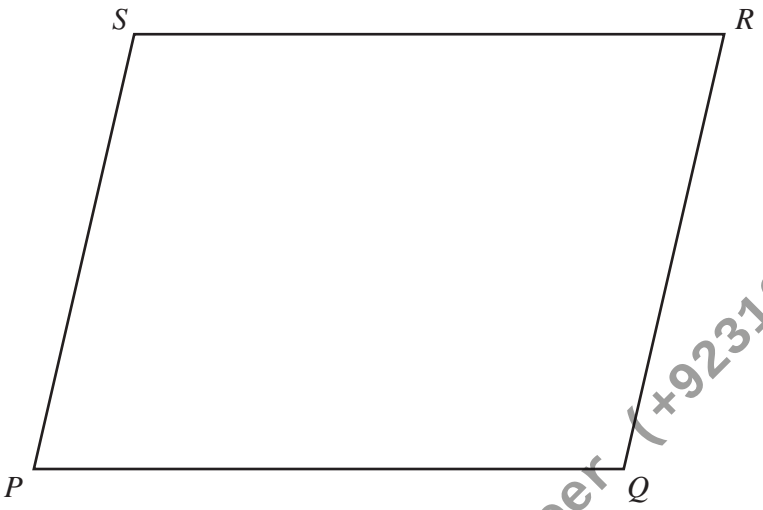
18 The diagram below shows the quadrilateral *PQRS*.

(a) On the diagram, construct

(i) the bisector of $S\hat{P}Q$, [1]

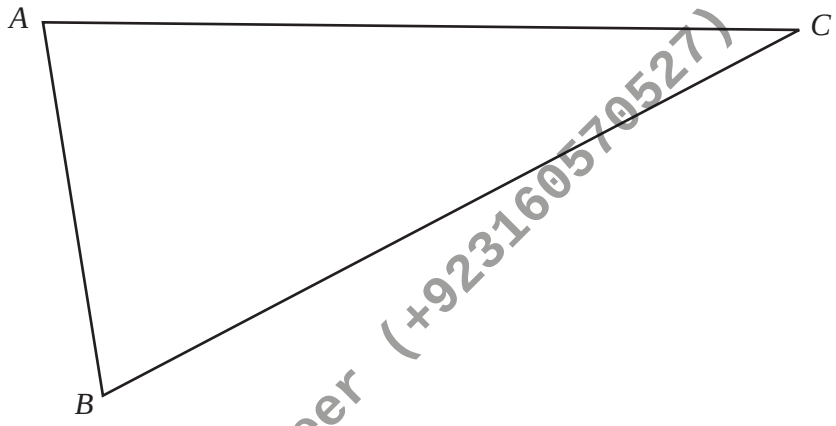
(ii) the perpendicular bisector of *QR*. [1]

(b) On the diagram, shade the region inside the quadrilateral containing the points that are closer to *PQ* than to *PS* and nearer to *Q* than to *R*. [1]



Q17

26 The diagram below shows triangle *ABC*.



(a) The point *D* is on the opposite side of *AC* to *B*.
AD = 6 cm and *CD* = 8 cm.

Construct triangle *ADC*. [1]

(b) On the diagram, construct the locus of points **inside** the quadrilateral *ABCD* that are

(i) 2.5 cm from *AC*, [1]

(ii) equidistant from *AB* and *BC*. [1]

(c) The points *P* and *Q* are 2.5 cm from *AC* and equidistant from *AB* and *BC*.

Mark and label *P* and *Q* .

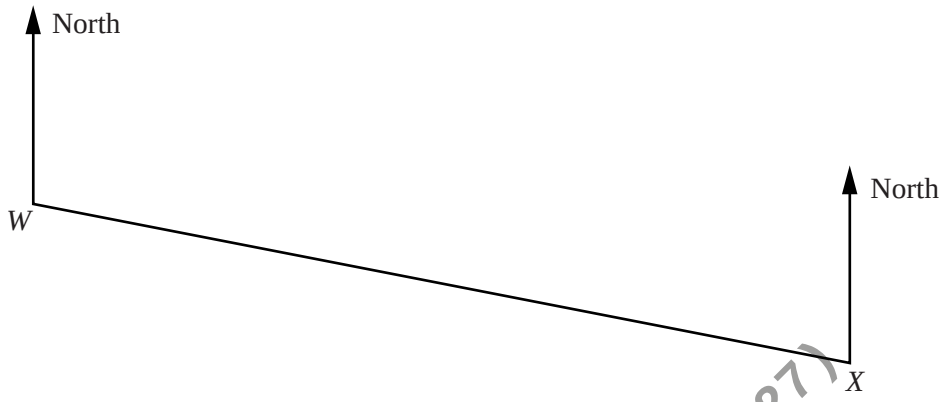
Measure *PQ*.

Answer *PQ* = ... cm [1]

Q18

4 The scale diagram shows the position of two hotels, *W* and *X*, drawn to a scale of 1 cm to 5 km.

Do not write in this margin



(a) Find, by measurement, the bearing of *W* from *X*.

Answer ..... [1]

(b) Hotel *Y* is 40 km from *W* and 37 km from *X*.

Given that *Y* is the furthest south, construct the position of *Y* on the diagram. [2]

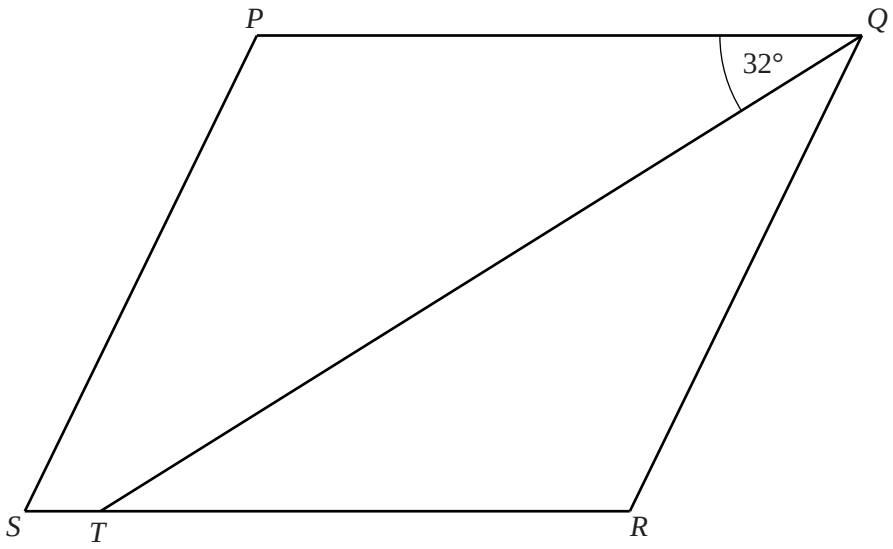
(c) The bearing of hotel *Z* from *W* is 072° and *Z* is due North of *X*.

By making an accurate drawing, mark the position of *Z* on the diagram.
Find the actual distance, in kilometres, between *Z* and *X*.

Answer ..... km [3]

Q19

(b)



PQRS is a parallelogram.
QT is the bisector of $P\hat{Q}R$ and $P\hat{Q}T = 32°$.

(i) Giving a reason for your answer, find

(a) $Q\hat{T}R$,

Answer $Q\hat{T}R$ = ... because .....

..... [1]

(b) $S\hat{P}Q$.

Answer $S\hat{P}Q$ = ... because .....

..... [1]

(ii) On the diagram, construct the locus of points inside the parallelogram *PQRS* which are

I 4 cm from *PS*,
II 5 cm from *R*. [2]

(iii) The point *V* is inside *PQRS*,
less than 4 cm from *PS*,
less than 5 cm from *R*,
nearer to *QR* than *PQ*.

Shade the region containing the possible positions of *V*. [1]

Q20

27 The diagram at the bottom of the page shows the lines AB and BC.

For Examiner's Use

(a) Measure $A\hat{B}C$.

Answer ..... [1]

(b) The point D is above AB.
AD and CD are each equal to AB.
On the diagram, construct quadrilateral $ABCD$. [1]

(c) On the diagram, construct the locus of points, **inside** the quadrilateral $ABCD$, that are

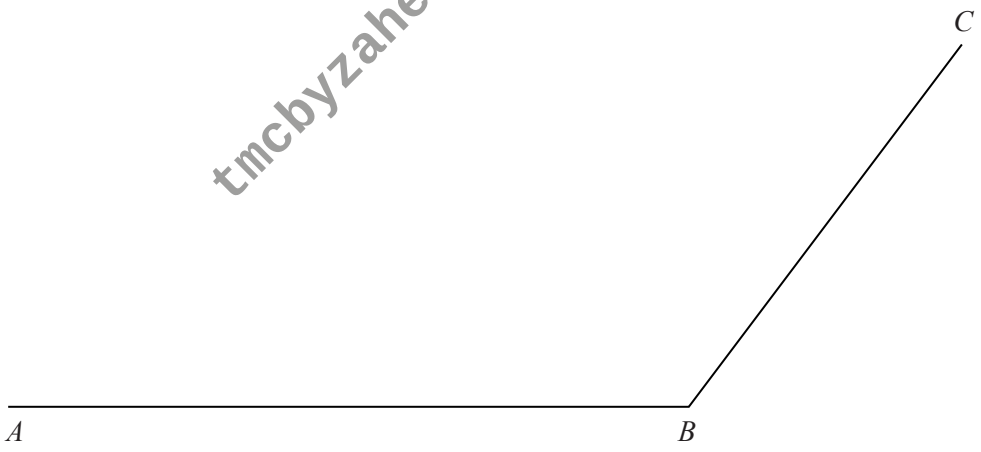
(i) 7 cm from C, [1]

(ii) equidistant from AB and BC. [1]

(d) These two loci meet at the point P.

Label the point P on the diagram and measure DP.

Answer DP = ... cm [1]



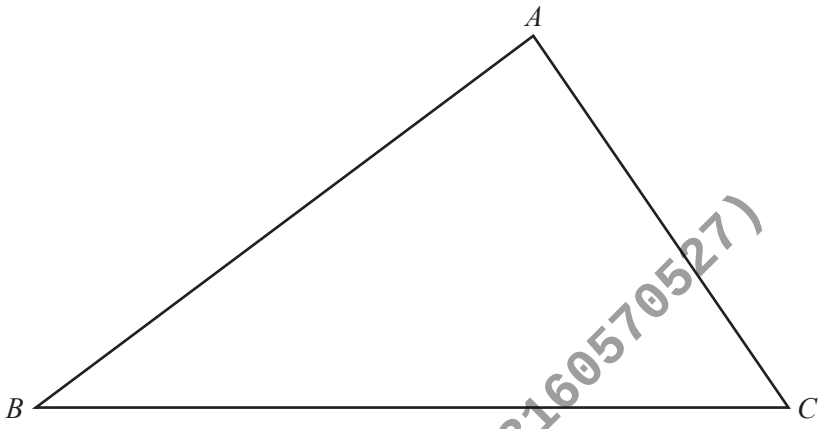
Q21

12 The diagram below shows triangle ABC.

For Examiner's Use

(a) Construct the perpendicular bisector of AB. [1]

(b) Shade the region inside the triangle containing points that are closer to A than to B and **more than** 6 cm from C. [2]



Section B [48 marks]

Answer **four** questions in this section.

Q22

Each question in this section carries 12 marks.

For Examiner's Use

8 The scale diagram shows the positions, A and B, of two buoys.
B is due South of A and $AB = 1500$ m.

(a) Write down the scale of the diagram.

Answer 1 cm to .....m [1]

(b) A third buoy is positioned at C which is due East of B and 1800 m from A.

Mark the position of C on the diagram. [2]

(c) **Calculate** the actual distance BC.
Give your answer correct to the nearest metre.

Answer .....m [2]

Q23

24 The diagram at the bottom of the page shows triangle ABC.

For Examiner's Use

(a) Measure $B\hat{A}C$.

Answer ..... [1]

(b) On the diagram, construct the locus of points, **inside** the triangle ABC, that are

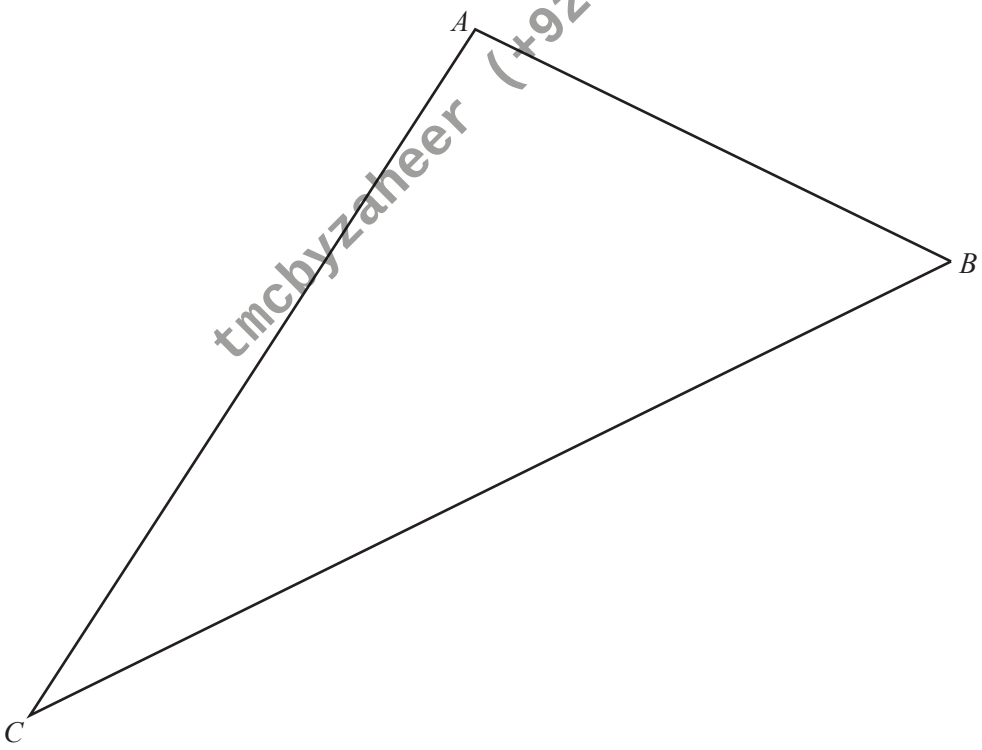
(i) equidistant from A and B, [1]

(ii) equidistant from AB and BC. [1]

(c) These two loci meet at the point P.

Label the point P on the diagram and measure CP.

Answer CP = ... cm [1]

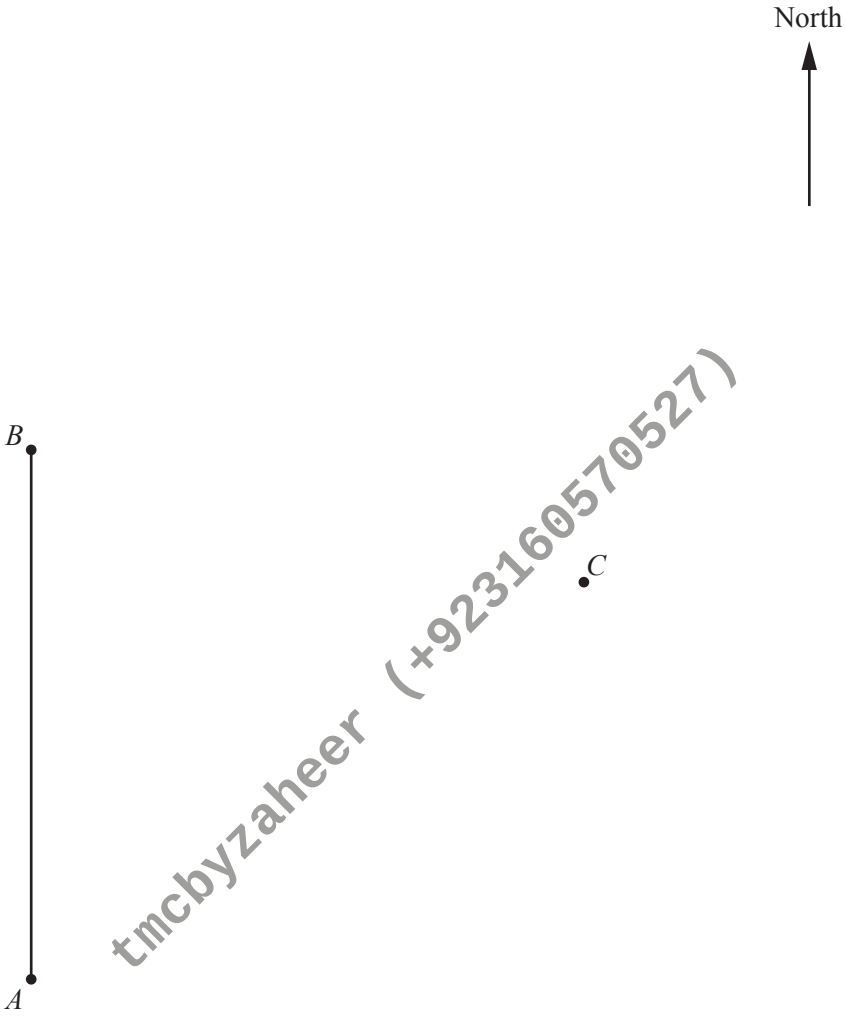


Section A [52 marks]

Answer **all** questions in this section.

Q24

1 The scale drawing shows three airfields, *A*, *B* and *C*, with *B* due north of *A*.
The scale is 1 cm to 20 km.



(a) Find the actual distance between *A* and *B*.

Answer ..... km [1]

(b) A beacon, D, is to the west of the line AB.
It is 100 km from A and 120 km from B.

Construct the position of D on the scale drawing. [2]

(c) Measure the bearing of C from B.

Answer ..... [1]

(d) An aircraft is

- equidistant from A and C,
- 90 km from B.

(i) By constructing suitable loci, mark on the diagram the two possible positions, P and Q, of the aircraft. [3]

(ii) Given that the aircraft is east of the line AB, find, by measuring, its bearing from C.

Answer ..... [1]

Q25

24 The diagram shows the positions, on a map, of three boats A, B and C.
The map has a scale of 1 cm to 1 km.

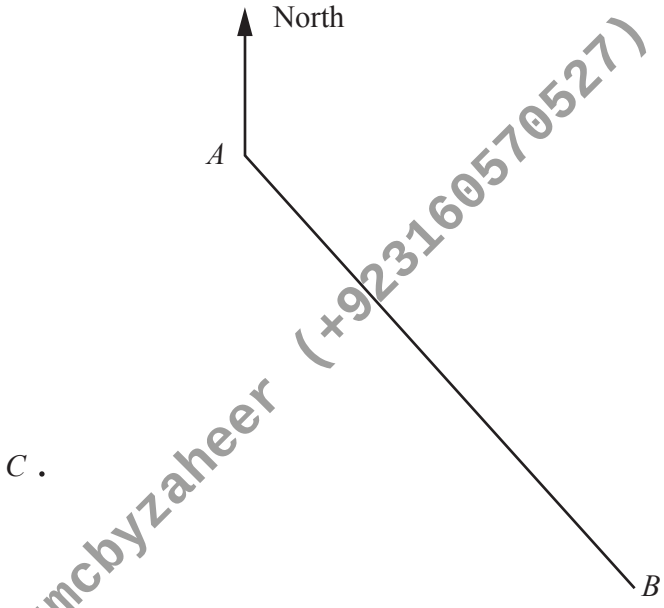
(a) Find the bearing of B from A.

Answer ..... [1]

(b) A fourth boat, D, is

- closer to B than to A,
- less than 4 km from C.

By drawing appropriate loci find, and shade, the region in which D is situated. [3]



Q26

6 $ABCD$ is a field in the shape of a trapezium.
$A\hat{B}C = 56°$, $B\hat{A}D = 104°$ and the distance between the parallel sides of the field is 90 m.

(a) Using a scale of 1 cm to 20 m, draw a plan of the field.
AB has been drawn for you.

[4]

(b) Find the actual distance CD.

Answer CD = ... m [2]

(2015 to June 2018 questions)

14 In triangle ABC, $AB = 5\,\text{cm}$ and $AC = 6\,\text{cm}$.

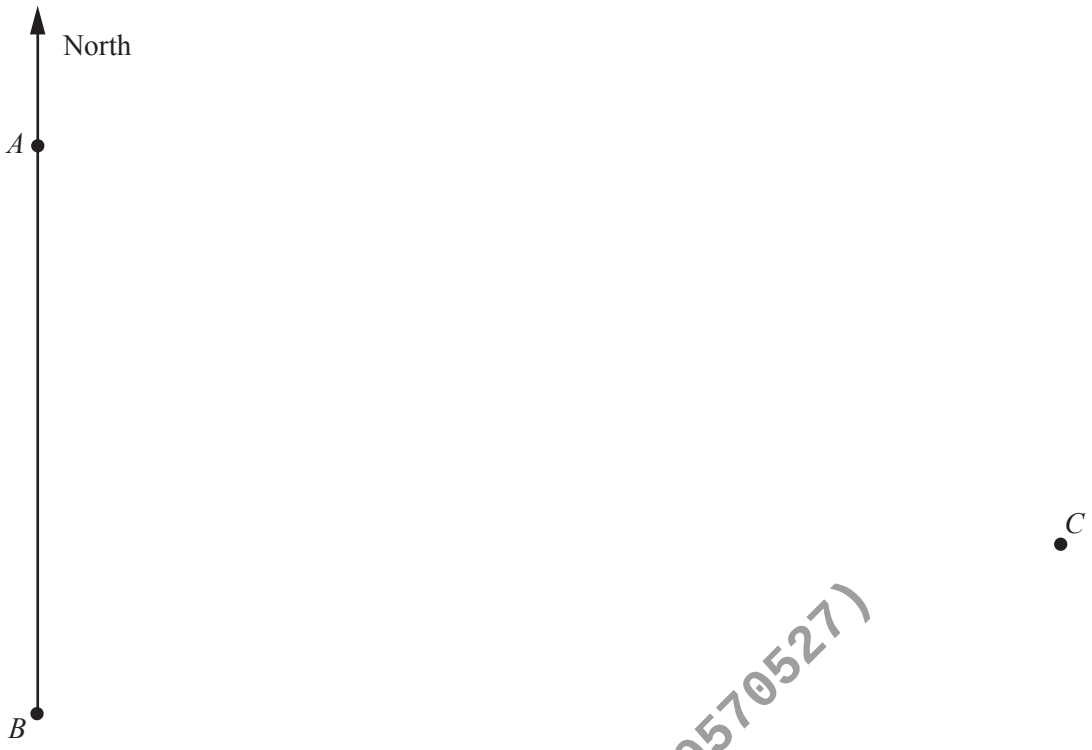
(a) Construct triangle ABC.
Line BC is drawn for you.

[2]

(b) Measure $B\hat{A}C$ in your triangle.

Answer ..... [1]

(d) The scale drawing, drawn to a scale of 1 cm to 2 km, shows A, B and a third port, C.



(i) When the boat has travelled 24 km, it stops at the point X.

Mark and label X on the diagram. [2]

(ii) A second boat is located

I less than 12 km from A
II nearer to BC than to BA.

Shade the region in which this second boat must lie. [3]

(iii) The point Y is the position of the second boat when it is as far as possible from X.

Mark and label Y on the diagram and hence find the maximum possible distance between the two boats.

Answer ..... km [2]

21 The diagram shows the positions of three ships A, B and C.
It is drawn to a scale of 1 cm to 20 km.

(a) Find, by measurement, the bearing of C from A.

Answer ..... [1]

(b) On the diagram construct the locus of points, inside triangle ABC, that are

(i) equidistant from B and C, [1]

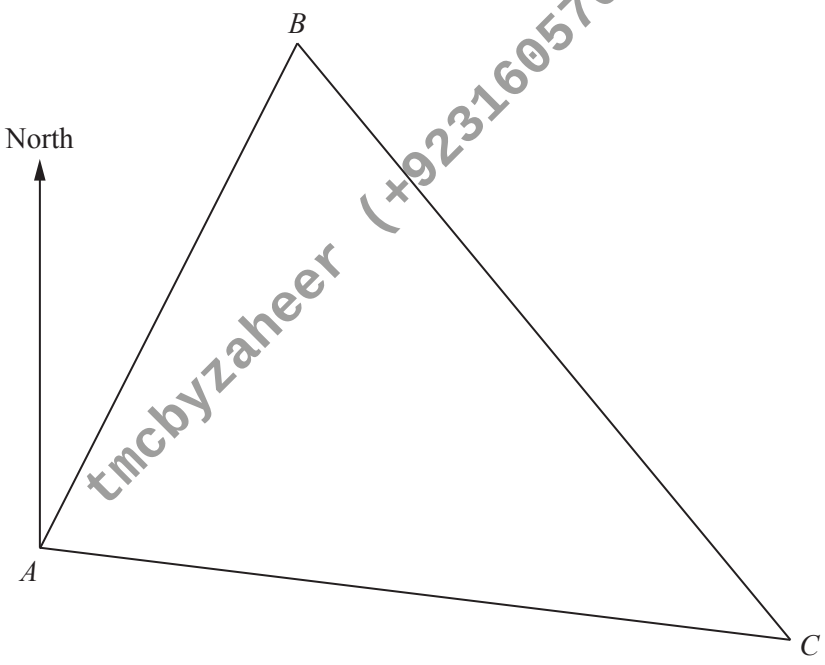
(ii) equidistant from AB and BC. [1]

(c) A ship D is
- equidistant from B and C,

and
- equidistant from AB and BC.

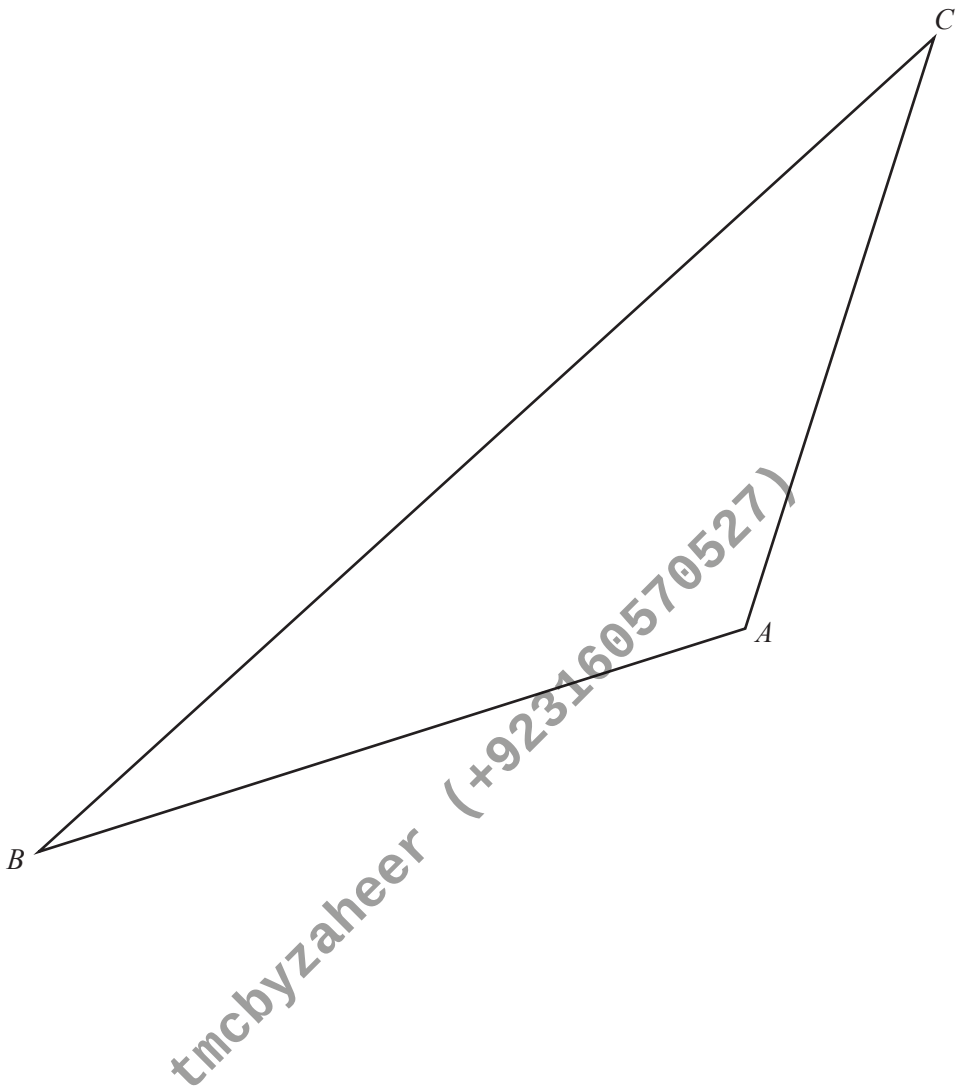
Label the position of D on the diagram and find the actual distance of D from A.

Scale: 1 cm to 20 km



Answer DA = ... km [1]

12 The diagram below shows triangle ABC.



(a) On the diagram construct the locus of points inside the triangle that are

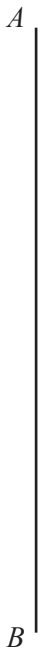
(i) 3.5 cm from A, [1]

(ii) equidistant from AC and BC. [1]

(b) On the diagram, shade the region inside the triangle containing the points that are more than 3.5 cm from A **and** closer to AC than to BC. [1]

4 **(a)** Triangle ABC has sides $AB = 8$ cm, $AC = 7$ cm and $BC = 12$ cm.

(i) Use a ruler and compasses to construct triangle ABC.
Side AB has been drawn for you.



[2]

(ii) Measure $B\hat{A}C$.

Answer ..... [1]

(b) Calculate the interior angle of a regular 12-sided polygon.

Answer ..... [2]

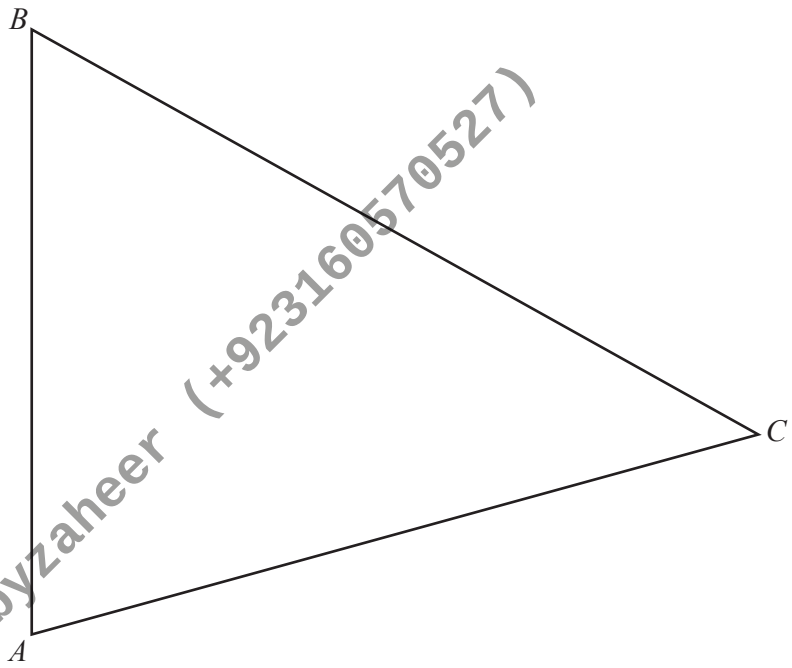
22 The diagram shows triangle ABC.

(a) Construct the locus of points, **inside** triangle ABC, that are equidistant from A and B. [1]

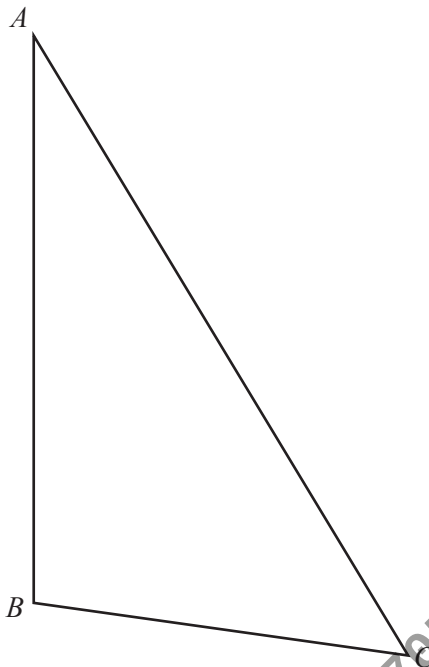
(b) Construct the locus of points, **inside** triangle ABC, that are equidistant from AB and BC. [1]

(c) On the diagram, shade the region **inside** triangle ABC which contains the points that are

nearer to A than to B and
nearer to BC than AB. [1]



23



The diagram shows the triangle ABC.

(a) Measure angle ABC.

Answer ..... [1]

(b) On the diagram, construct the perpendicular bisector of AB. [1]

(c) On the diagram, construct the locus of points that are 5 cm from C. [1]

(d) The points P and Q lie on the perpendicular bisector of AB and are 5 cm from C.

Mark and label the points P and Q on the diagram and measure PQ.

Answer PQ = ... cm [1]

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The diagram shows the position of point A.
Point B is 8 cm from A on a bearing of 062°.
Point C is 6.5 cm from A on a bearing of 194°.

(a) (i) Find and label B and C. [3]

Point D is the point on BC that is the shortest distance from A.

(ii) Find and label D. [1]

(iii) Measure AD.

Answer ..... cm [1]

(iv) By taking measurements, find the ratio $CD : DB$.
Give your answer in the form $1 : n$.

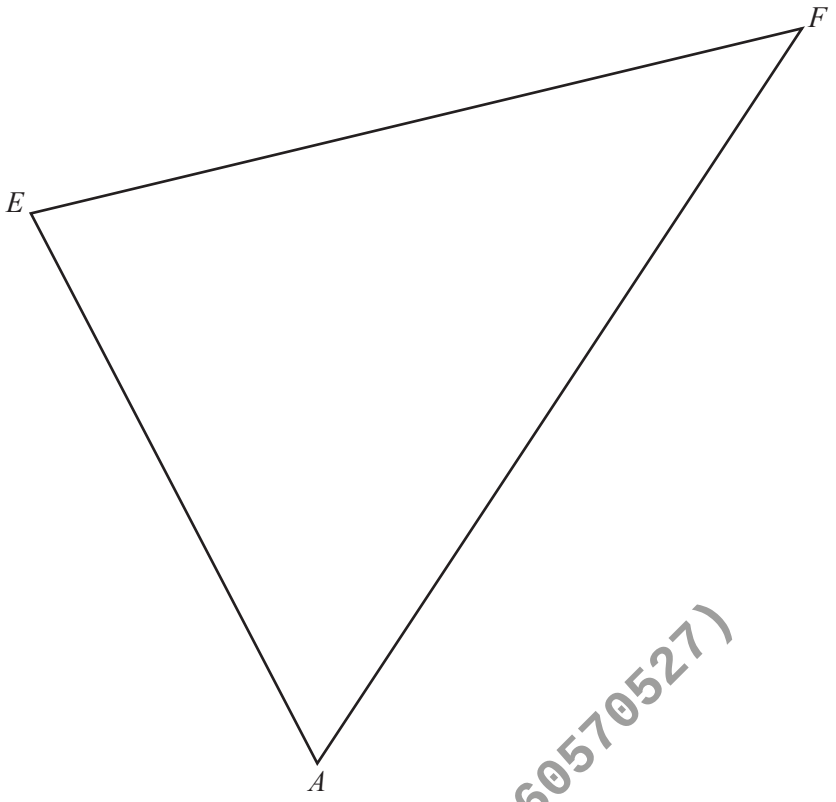
Answer 1 : ..... [2]

(v) The area of triangle ADB is w cm^2.

Giving your answer in terms of w, find the area of triangle ADC.

Answer ..... cm^2 [1]

(b)

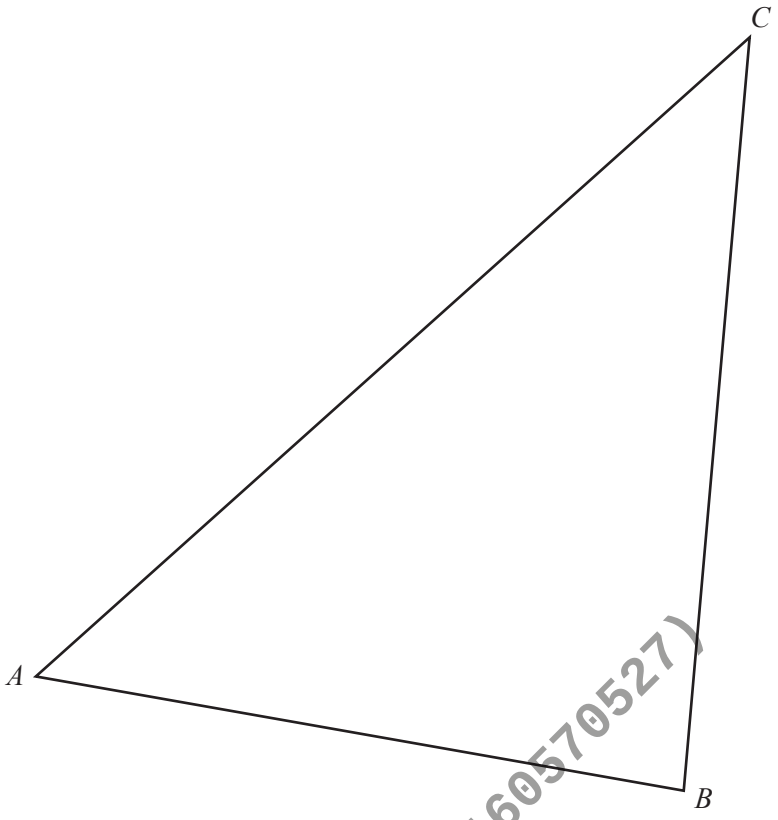


The diagram shows the positions of A, E and F.

Construct and shade the region inside triangle AEF that is

- less than 6 cm from E
- nearer to AF than to AE
- nearer to A than to F. [4]

19



(a) On the diagram, construct the perpendicular bisector of AB. [1]

(b) On the diagram, construct the locus of points inside triangle ABC, that are

(i) 7 cm from C, [1]

(ii) equidistant from AB and AC. [1]

(c) P is any point which is

equidistant from A and B
and more than 7 cm from C
and nearer to AC than AB.

Find the extremes of the possible positions of P and label them P_1 and P_2. [1]